

OLLSCOIL NA GAILLIMHE
UNIVERSITY OF GALWAY
SCHOOL OF ENGINEERING

ENGINEERING MATHS QUALIFYING EXAMINATION 2024

First Paper

Time allowed: *Two* hours

Candidates for Computer Science & Information Technology and Project & Construction Management should take 4 questions out of 6. All other candidates should take 5 questions out of 6.

Formulae and Tables booklets are provided by the Exams Office
Calculators are permitted

1. (a) Solve the simultaneous equations

$$\begin{aligned}\log_2(x) + \log_2(y) &= 6 \\ x + y &= 34\end{aligned}$$

for $x, y > 0$.

- (b) Find the set of values for which $|x^2 - 4| < 1$.
- (c) A container in the shape of an inverted cone of radius r and perpendicular height h is filled with water. This water is then poured into a similar conical container of radius $2r$ and height $2h$; a spherical solid of radius $r/4$ is then submerged in the water. Derive a formula for the height of the empty space above the water in the second container.
2. (a) $m + 10$, m and $2m - 21$ are the first three terms in a geometric progression, where $m > 0$. Determine the value of m , and hence calculate the sum to infinity of the series.
- (b) An arithmetic progression begins with $x + 11$, $4x + 4$, $9x + 5$, where $x \in \mathbb{Z}$. Find the sum of the first 11 terms.
- (c) In an arithmetic progression, the sum to 10 terms is numerically equal to the first term, and the seventh term is numerically equal to the common difference. What is the first positive term of the progression?
- (d) A saver puts €1000 in a savings account paying interest of 2% every three months. After how many years will the value of the savings have doubled?
3. (a) Given that θ is an acute angle, find the *exact* value of $\tan \theta$ if $\sin^2 \theta = 3/4$.

- (b) Solve $6 \cos^2 x + \sin x = 5$ for $0 \leq x \leq 360^\circ$, giving your answers to one decimal place.
- (c) A cuboid has length x cm, width $2x$ cm and height $3x$ cm. The cuboid is expanding uniformly due to being heated. If A is the surface area of the cuboid and V is its volume, find $\frac{dA}{dx}$ and $\frac{dV}{dx}$, and use them to show that if $\frac{dV}{dt} = 3 \text{ cm}^3/\text{s}$ then $\frac{dA}{dt} = \frac{22}{3x} \text{ cm}^2/\text{s}$.
4. (a) Differentiate the following functions with respect to x :
- $f(x) = \cos(x - \pi)$;
 - $g(x) = x^2 e^x$;
 - $h(x) = \frac{\sin x}{x}$.
- (b) Consider the curve of the function $f(x) = \ln(1 + x^3)$, defined for $x > -1$.
- Identify the unique point P where the slope of the curve is zero;
 - show that the slope of the curve is positive at all points other than P ;
 - identify both points of the curve where the second derivative is zero;
 - find the equation of the tangent to the curve at the point where $x = 1$.
5. (a) Integrate the following:
- $\int \frac{1}{5 - 3x} + 2e^{x/9} - 4 \sin(6x) \, dx$;
 - $\int 4x \left(\frac{7x^4 - 5x^{-3}}{x^2} - \sqrt{x} \right) dx$.
- (b) Show that $\int_0^1 \frac{x}{\sqrt{x^2 + 2}} dx = \sqrt{3} - \sqrt{2}$.
- (c) Determine the area bounded between the curves $y = 2x(4 - x)$ and $y = x^3$.
6. (a) Simplify $\frac{1+i}{i-1} + \frac{1-i}{i+1}$.
- (b) Using de Moivre's theorem, find in polar form all four solutions to the equation $z^4 = 16$.
- (c) By deriving the equivalent cartesian equations, show that the equation $|z - 3i| = 2$ represents a circle on the Argand diagram while $|z - 3i| = |z - 3|$ represents a straight line. Explain why the circle and line do not intersect.

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Second Paper

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1. (a) To convert a temperature c in degrees celsius to f in degrees fahrenheit we use the formula $f = \frac{9}{5}c + 32$. Calculate the numerical value of the temperature that is the same in both systems.

- (b) Solve the following system of simultaneous equations:

$$3x - y + 2z = -3$$

$$x + 2y - 3z = 0$$

$$2x - 3y + z = -7$$

- (c) Water is leaking from a closed cuboid tank at a rate of 1 litre per minute, and is collected in a cylindrical bucket underneath. If the cuboid tank is 3m long, 2m wide and 1m high and is initially full, and the radius and height of the cylinder are both 50cm, how many times will the bucket be filled before the tank empties, and how long to the nearest hour will this process take?

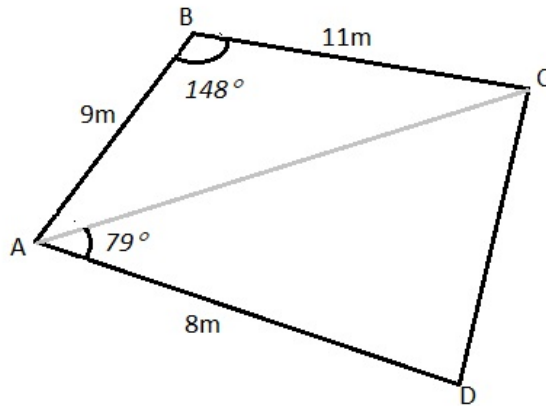
2. (a) How many distinct arrangements are there of the letters of the word KANGAROOS:

- i. if there are no restrictions;
- ii. if both A's appear side by side;
- iii. if vowels must be separated;
- iv. if consonants are grouped together?

If four letters are chosen at random from the word KANGAROOS without replacement, what is the probability that they will spell out the word ROOK?

- (b) Find the number of ways in which six different books can be placed in two boxes, A and B, if each box is to get at least two books.

3. (a) Show that $(x + y)$ is a factor of $2x^3 + 7x^2y + 7xy^2 + 2y^3$, and factorise the expression completely.
- (b) Calculate the area of the quadrilateral $ABCD$ in the diagram below.



4. (a) Find the equation of the circle having the line segment joining the points $(2, 1)$ and $(4, 7)$ as diameter, and find the equations of the tangents from the point $(3, 0)$ to this circle.
- (b) Prove that the circles

$$x^2 + y^2 - 20x - 14y + 113 = 0$$

$$4x^2 + 4y^2 + 16x - 16y - 49 = 0$$

lie entirely outside each other and find the length of the shortest distance from a point on one circle to a point on the other.

5. (a) Suppose 70% of all candidates who sit a certain exam will pass, while the remainder fail. An examiner marks six complete exam scripts one at a time then records a pass or fail outcome. What is the probability that:
- The first script is a pass and the next five fail?
 - the first script is a fail and the next five pass?
 - exactly four pass?
 - the first fail occurs on the fourth script?
- (b) Suppose the marking scheme is revised so that an extra 'Query' grade is introduced, with the effect that 20% receive a query grade and now only 40% achieve a pass with remainder failing. If six scripts are marked, what is the probability that:
- all the scripts fail?
 - more than half the scripts get a query grade?
 - there are two passes, two fails and two query grades?

6. (a) Solve the equation

$$3 - \cos 2x = 7 \cos x$$

for all values of x in the interval $0 \leq x \leq 2\pi$.

- (b) If x, y, z are all positive and $x + y + z = 1$, show that $1/x + 1/y + 1/z > 3$.
- (c) A square is inscribed inside a semicircle such that one corner lies on the midpoint O of the semicircle's base while the opposite corner lies on the circumference, as shown in the diagram below. If the area of the square is 5cm^2 , find the length of the chord BC .

