

**OLLSKOIL NA GAILLIMHE**  
**UNIVERSITY OF GALWAY**  
**SCHOOL OF ENGINEERING**

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*ENGINEERING MATHS QUALIFYING EXAMINATION 2023*

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*First Paper*

Time allowed: *Two* hours

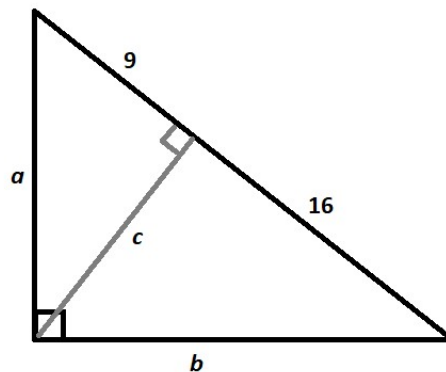
Candidates for Computer Science & Information Technology and Project & Construction Management should take **4** questions out of 6. All other candidates should take **5** questions out of 6.

**Formulae and Tables booklets are provided by the Exams Office**  
**Calculators are permitted**

1. (a) Solve the simultaneous equations

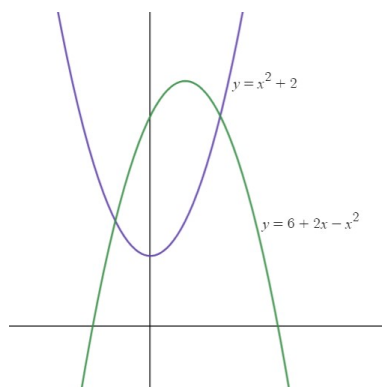
$$\begin{aligned}\log_2(xy) &= 7 \\ \log_2\left(\frac{x^2}{y}\right) &= 5\end{aligned}$$

- (b) Find the set of values for which  $|x^2 - 9| < 8$ .
- (c) A perpendicular is drawn from the right angle to the hypotenuse of a right-angled triangle; this divides the hypotenuse into lengths 9 and 16 units as depicted in the diagram below. Using this information, determine the lengths  $a$ ,  $b$  and  $c$  in the diagram.



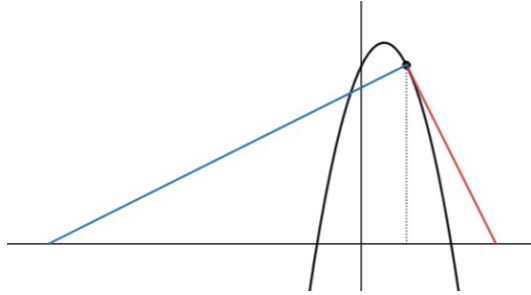
2. (a) The sum  $S_n$  of the first  $n$  terms of a series is  $1 - (\frac{1}{4})^n$ .
- i. By obtaining the values of the first three terms of this series, show that it is a geometric progression.

- ii. Hence or otherwise calculate the sum to infinity.
- (b) An arithmetic progression is such that the sum of the first  $n$  terms is  $2n^2$  for all positive integral values of  $n$ . Find, by substituting two values of  $n$  or otherwise, the first term and the common difference.
- (c) In an arithmetic progression, the 8th term is twice the 3rd term and the 20th term is 110.
- Find the common difference;
  - Calculate the sum of the first 100 terms.
- (d) A saver puts €1000 in a savings account paying interest of 1.5% every six months. What will the account hold at the end of five years? Calculate the equivalent annual interest rate.
3. (a) Simplify  $\frac{5(1-2i)(5+i)}{4-3i} + 3i$ .
- (b) Using de Moivre's theorem, find all five solutions to the equation  $z^5 = 1$ .
- (c) i. Show that  $z - 2$  is a factor of  $z^4 - z^3 - z^2 - z - 2$ ;  
 ii. by finding the other factors, or otherwise, derive the four solutions to  $z^4 - z^3 - z^2 - z - 2 = 0$ .
4. (a) Determine the following indefinite integrals.
- $\int -7e^{x/2} - 3\cos(4x) + \frac{1}{5x-3} dx$
  - $\int \sqrt{2}x\left(\frac{7x^3-5}{x^2} - x\right) dx$
- (b) Determine the area bounded between the curves  $y = x^2 + 2$  and  $y = 6 + 2x - x^2$ .



5. (a) Use a trigonometric identity to show that  $\cos 105^\circ = \frac{1 - \sqrt{3}}{2\sqrt{2}}$ .
- (b) Prove that  $\frac{\cos \theta}{1 - \tan \theta} + \frac{\sin \theta}{1 - \cot \theta} = \sin \theta + \cos \theta$ .
- (c) Water is dripping from a hole in the base of a cylinder of radius  $r$  cm, where the water height is  $h$  cm, at a rate of 0.3 cubic centimetres per second.

- i. Find an expression for  $\frac{dh}{dt}$ , the rate at which the water level is falling in the cylinder.
  - ii. Hence find the rate of change in the water level, per minute, in a cylinder of radius 6 cm when the height of the water is 4 cm.
6. (a) Differentiate the following functions with respect to  $x$ :
  - i.  $f(x) = \sin \pi x$ ;
  - ii.  $g(x) = (x - 2)(\ln x)$ ;
  - iii.  $h(x) = \frac{1 + x + x^2}{1 - x - x^2}$ .
- (b) Let  $a$  and  $b$  be real numbers. The equation  $\frac{d^2y}{dx^2} + a^2y = 0$  is very important in engineering; show that  $y = \sin(ax + b)$  is a solution to this equation.
- (c) Consider the curve  $y = (4 - x)(x + 2)$ . Show that the triangular area bounded by the tangent and normal to the curve at the point  $(2, 8)$  and the x-axis is 80 square units.



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*ENGINEERING MATHS QUALIFYING EXAMINATION 2023*

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*Second Paper*

Time allowed: *Two* hours

Candidates for Computer Science & Information Technology and Project & Construction Management should take 4 questions out of 6. All other candidates should take 5 questions out of 6.

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1. (a) The points  $A(1, -6)$ ,  $B(2, 1)$  and  $C(5, 2)$  all lie on the circle  $S$ .
  - i. Find the equation of the perpendicular bisector of the line segment  $AC$ ;
  - ii. find the equation of the perpendicular bisector of the line segment  $AB$ ;
  - iii. hence or otherwise derive the equation of  $S$ ;
  - iv. determine the point of intersection of the tangents to  $S$  at  $A$  and  $B$ .

2. (a) i. Use a trigonometric identity to express

$$4 \sin \theta - 3 \cos \theta$$

in the form  $R \sin(\theta - \alpha)$ , where  $R$  is positive and  $\alpha$  is an acute angle.

- ii. Hence or otherwise find the greatest and least values of the expression

$$\frac{1}{10 - 3 \cos \theta + 4 \sin \theta}$$

- (b) A fuel tank is constructed from a cylinder of metal of negligible thickness, closed at one end and with a hemispherical section of the same material at the other end. The radius of the hemisphere and the cylinder is 0.5 m, while the length of the cylinder is 2 m. Once filled, the tank develops a leak and loses fuel at a constant rate of 1 litre per minute. To the nearest minute, how many days, hours and minutes will it take for the tank to empty?
3. (a) An exam paper consists of 30 questions. For each correct answer four marks are given, and for each incorrect answer three marks are deducted. A student answers all 30 questions, getting a total score for the exam of 71 marks. How many questions did the student answer correctly?

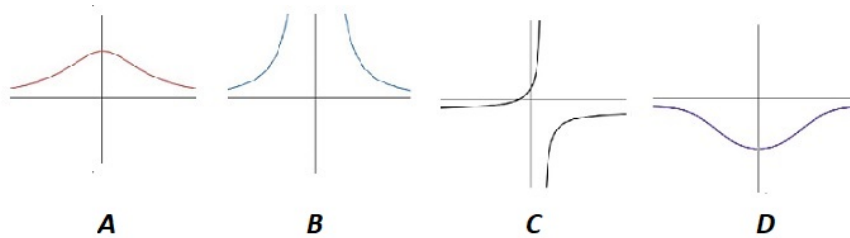
(b) Solve the following system of simultaneous equations:

$$2x - y + 4z = 12$$

$$x + y + 2z = 9$$

$$3x + y - 5z = -10$$

(c) Which one of the following graphs represents the function  $f(x) = \frac{1}{1+x^2}$ ? Give a reason for your answer.



4. (a) How many distinct arrangements are there of the letters of the word PLATYPUS:
- if there are no restrictions;
  - if both P's appear side by side;
  - if vowels must be separated;
  - if consonants are grouped together?

If four letters are chosen at random from the word PLATYPUS without replacement, what is the probability that they will spell out the word PAST?

- (b) Find the number of ways in which eight different books can be placed in two boxes, A and B, if each box is to get at least two books.
5. (a) Suppose 70% of all candidates who sit a certain exam will pass. An examiner marks six complete exam scripts one at a time then records a pass or fail outcome. What is the probability that:
- The first script is a pass and the next five fail?
  - the first script is a fail and the next five pass?
  - exactly four pass?
  - the first fail occurs on the fourth script?
- (b) Suppose the marking scheme is revised so that an extra 'Query' grade is introduced, with the effect that 10% receive a query grade and now only 50% achieve a pass. If six scripts are marked, what is the probability that:
- all the scripts fail?

- ii. more than half the scripts get a query grade?
  - iii. there are two passes, two fails and two query grades?
6. (a) Show that  $(x - y)$  is a factor of  $2x^3 - 3x^2y + y^3$ , and factorise the expression completely.
- (b)  $ABC$  is a triangle with  $|AB| = 4$  cm,  $|AC| = 5$  cm and angle  $\angle ACB = 30^\circ$ .
- i. Calculate the two possible values for the length  $|BC|$ , giving your answer correct to three significant figures;
  - ii. for each of these lengths, calculate the corresponding values for the angle  $\angle ABC$  correct to the nearest tenth of a degree.