OLLSCOIL NA hÉIREANN, GAILLIMH

NATIONAL UNIVERSITY OF IRELAND, GALWAY

COLLEGE OF ENGINEERING AND INFORMATICS

ENGINEERING MATHS QUALIFYING EXAMINATION 2022

First Paper

Time allowed: *Two* hours

Candidates for Computer Science & Information Technology and Project & Construction Management should take **4** questions out of 6. All other candidates should take **5** questions out of 6.

Formulae and Tables booklets are provided by the Exams Office
Calculators are permitted

1. (a) A particle moves in a straight line, and its displacement s metres in the positive direction from an origin O as a function of time t seconds is given by $s(t) = t^3 - 7t^2 + 14t - 8$. Consider its *first four seconds* of motion:

   i. Determine the initial velocity and displacement v_0 and s_0 respectively;

   ii. Show that the particle passes through O at $t = 4$;

   iii. Determine the average velocity;

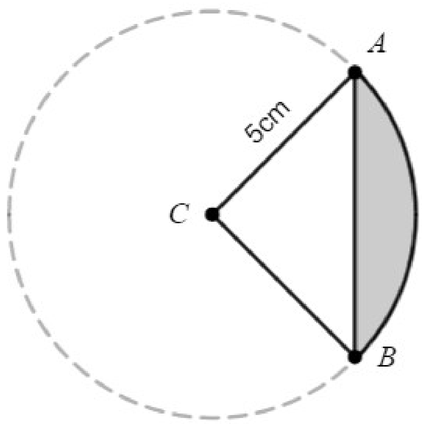
   iv. Calculate the times at which the particle reverses its direction of travel;

   v. Determine the time interval during which the particle's displacement is positive.

   (b) In the diagram below, AB is an arc of a circle centred at C. If the radius of the circle is 5 cm and the length of the arc AB is 2π cm, then

   i. Calculate the length of the *straight line segment* AB;

   ii. Calculate the shaded area.



2. (a) i. An arithmetic sequence begins $36, 32, 28, 24, \ldots$. Find the possible values of n for which $S_n = 176$.

ii. A convergent geometric sequence satisifes the relationship $8S_\infty = 9S_4$, where S_4 is the sum of the first four terms of the sequence and S_∞ is the sum to infinity. Determine the common ration in the form $1/\sqrt{a}$, where $a \in \mathbb{N}$.

(b) When a baby is born, €3000 is invested for her in an account with a fixed interest rate of 4% per year.

i. To the nearest cent, what will the account be worth at the start of the seventh year?

ii. After how many full years will the account have doubled in value?

3. (a) Let $u = 1 + \sqrt{3}i$ and $v = -1 - i$, where $a, b \in \mathbb{R}$ and $i^2 = -1$, and let $z = u/v$.

i. Write z in the form $r(\cos\theta + i\sin\theta)$, where $r, \theta \in \mathbb{R}$.

ii. Using de Moivre's theorem, or otherwise, find z^5.

(b) Consider the equation $9z^3 - 9z^2 + 4z - 4 = 0$

i. Show that $z = 1$ is a solution.

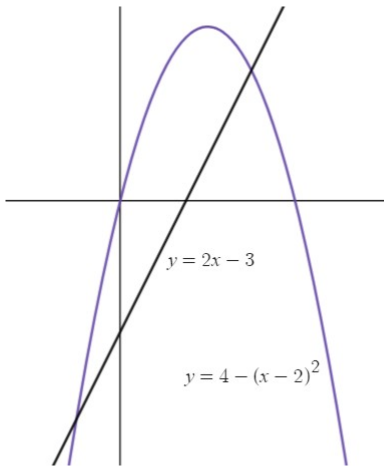
ii. Find the remaining solutions.

4. (a) Find the following indefinite integrals.

i. $\displaystyle\int 5e^{3x} - 2\sin(3x) + \frac{1}{2x-5}\, dx$

ii. $\displaystyle\int \sqrt{x}(x + \frac{1}{x})\, dx$

(b) Determine the area bounded between the curves $y = 4-(x-2)^2$ and $y = 2x-3$.



5. (a) Prove that $\dfrac{\sin\theta}{1+\cos\theta} = \tan\dfrac{\theta}{2}$.

(b) A snowball of radius r cm is melting uniformly in such a way that it maintains its spherical shape, but its radius decreases by 1.6cm per hour. If the surface area of the snowball at time t hours is $A\,\text{cm}^2$, find $\dfrac{dA}{dt}$ when $r = 5.5\,\text{cm}$. Give your answer to 2 d.p.

6. (a) Differentiate the following functions with respect to x:

    i. $f(x) = \sin\pi - 2\cos\frac{x}{2} + \sqrt{x-7}$;

    ii. $g(x) = (2 + x^2)(\tan x)$;

    iii. $h(x) = \dfrac{\log_e x}{e^x}$.

   (b) Show that $y = 3e^{x/2}$ is a solution to the equation $2\dfrac{d^2y}{dx^2} + \dfrac{dy}{dx} - y = 0$.

   (c) There are two points on the curve $y = \dfrac{1}{1+x}$ where the gradient of the tangent is -1. Find the coordinates of both of these points.

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Second Paper

Time allowed: *Two* hours

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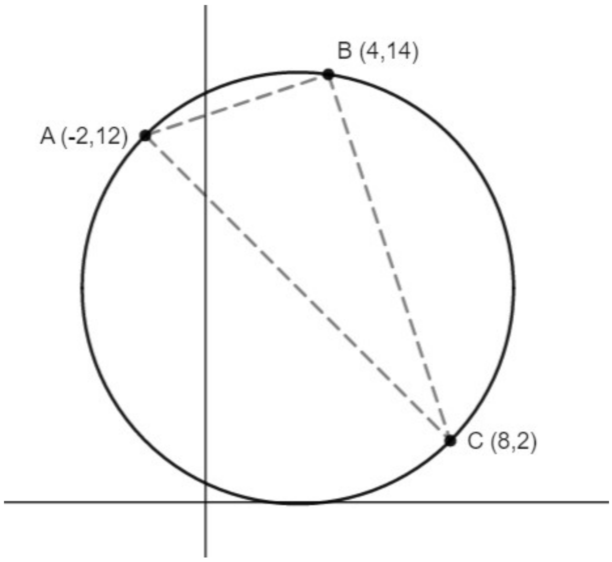
1. (a) The points $A(-2, 12)$, $B(4, 14)$ and $C(8, 2)$ all lie on the circle s as shown below:



i. Prove that the line segment AC is a diameter of the circle;

ii. Hence or otherwise derive the equation of the circle.

(b) Consider the circle $s : x^2 + y^2 + 2x + 4y - 21 = 0$ as well as the point $P(5, 6)$ outside s.

i. Find the shortest distance from P to s;

ii. find the length of each tangent from P to s.

2. (a) Three positive integers a, b and c form a *Pythagorean Triple* if $a^2 = b^2 + c^2$ is true. If $a = m^2 + n^2$, $b = 2mn$ and $c = m^2 - n^2$ where m, n are positive integers such that $n < m$, show that a, b and c form a Pythagorean Triple.

(b) Find all values of x for which $e^x - 6e^{-x} + 1 = 0$.

(c) A cylindrical container, with base radius 8 cm and height 20 cm, is fi
lled with water to a height of 15 cm. Find greatest number of solid spheres of radius 2 cm that can be submerged in the water without causing the water to flow out of the top of the cylinder.

3. (a) Solve the following inequalities:

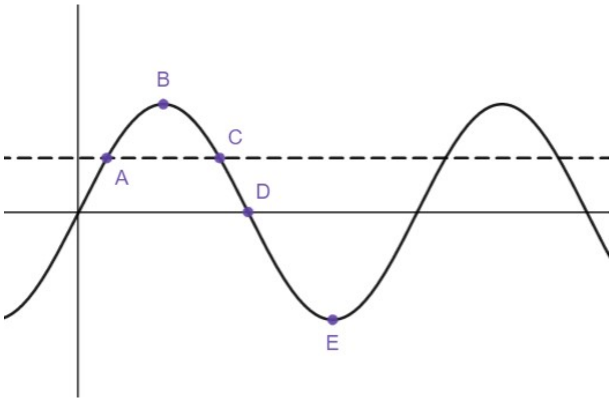
   i. $x^2 + x - 1 > 1$

   ii. $\dfrac{1}{x} < 1$

   iii. $|x - 3| < 4$

   (b) Find the values of c and δ that make the expression $|x - c| < \delta$ equivalent to $-4 < x < 2$.

   (c) A portion of the graph of $y = \sin(2x)$ is shown together with the coordinate axes; the dashed line is $y = 1/2$. Identify the points A, B, C, D and E.



4. (a) How many arrangements are there of the letters of the word WOMBAT:

   i. if there are no restrictions,

   ii. if O and M are side by side, in that order,

   iii. if O and M are separated,

   iv. if consonants are grouped together?

   If four letters are chosen at random from the word WOMBAT without replacement, what is the probability that they will spell out the word BOAT?

   (b) Five distinct points are arranged such that four of the points form the vertices of a square, with the centre of the square forming the fifth point. How many triangles can be formed using any three of the points as vertices?

5. Each night for a week, Florence and Dougal play a card game.

   (a) Suppose none of the games result in a draw, and the probability on any given night that Florence will win the game is 0.7. What is the probability that over the course of the seven days

   i. Dougal wins the first game and Florence wins the rest?

ii. Florence will win the the first, third, fifth and seventh games and Dougal wins the rest?

iii. Dougal wins exactly five games?

iv. Florence's first win will happen on the fifth day?

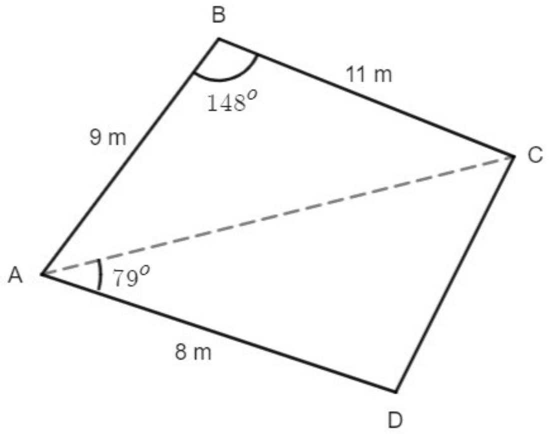
(b) Suppose instead that each game results in either a win or a draw, and the probability of a draw is 0.1, while the probability that Florence will win is now 0.5.

i. What is the probability that Dougal will win?

ii. What is the probability of four draws?

iii. What is the probability the series is tied with three wins for each player and one draw?

6. (a) Find the area of the quadrilateral $ABCD$ shown in the diagram below.



(b) The diagram below shows an isosceles triangle ABC; the angle at A is $42°$ and the side opposite A is of length $4\sqrt{2}$ units. It is readily verified that the length x of each of the equal sides is given by the expression $x = \dfrac{2\sqrt{2}}{\sin 21°}$.

i. Use the Cosine rule to show that $x = \dfrac{4}{\sqrt{1 - \cos 42°}}$;

ii. use the Sine rule to show that $x = \dfrac{4\sqrt{2}\sin 69°}{\sin 42°}$;

iii. use any of these expressions to determine the value of x to 2 d.p.