OLLSCOIL NA hÉIREANN, GAILLIMH

NATIONAL UNIVERSITY OF IRELAND, GALWAY

COLLEGE OF ENGINEERING AND INFORMATICS

ENGINEERING MATHS QUALIFYING EXAMINATION 2021

First Paper

Time allowed: *Two* hours

Candidates for Computer Science & Information Technology and Project & Construction Management should take **4** questions out of 6. All other candidates should take **5** questions out of 6.

Formulae and Tables booklets are provided by the Exams Office
Calculators are permitted

1. (a) i. If 13 and 49 are the third and seventh terms respectively in an arithmetic sequence then find the sum to 20 terms of the same sequence.

   ii. The numbers $\frac{t}{4}$, t and $4t$ are the first, third and fifth terms respectively in a geometric series for which the sum of the fourth and fifth terms is 30 and the common ratio is *positive*; find the value of t.

   iii. For this value of t, show that $8t$ is the sixth term of each of the sequences in parts (i) and (ii).

   (b) A snail is trapped at the bottom of a well, and starts to climb vertically upwards. Initially it climbs 50 centimetres, then needs to rest. It then climbs 25 centimetres before resting again. It carries on like this, each climbing session covering half the vertical distance of the previous session. Show that the snail will never escape if the well is more than one metre deep.

2. (a) An object travels in such a way that its displacement in metres from a fixed origin is represented by the function

$$s(t) = t^3 - 7t + 6$$

   for $t \geq 0$, where t is measured in seconds.

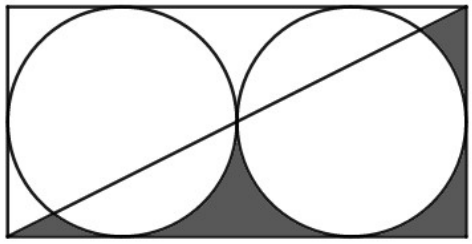
   i. What is the initial displacement?

   ii. At what time is the object's speed equal to -4m/s? What is the object's displacement at this instant?

   iii. Determine the time interval for which the displacement is negative.

   iv. At what time does the object reach its maximum negative displacement?

(b) A rectangle of width 20cm and height 10cm is just large enough to contain two touching circles as shown in the diagram below. Determine the combined area of the shaded regions.



3. (a) Differentiate the following functions with respect to x:

   i. $f(x) = e^{5x} + \cos x - \sqrt{x+1} + e^{\pi}$;

   ii. $g(x) = x^2 \log_e x$;

   iii. $h(x) = \dfrac{e^x}{\sin x}$.

   (b) Show that $y = \tan x$ is a solution to the equation $\dfrac{dy}{dx} = 1 + y^2$.

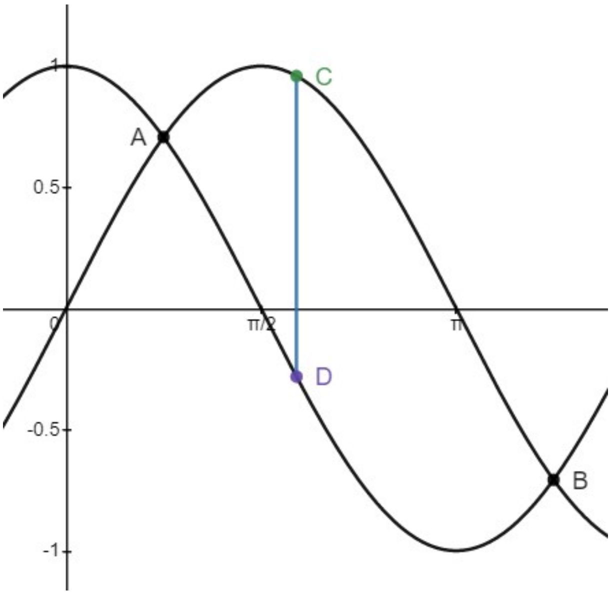
   (c) The normal to the parabola $y = x^2 + 2x - 1$ at the point $P(1, 2)$ crosses the x-axis at the point X. Find the area of the triangle OPX, where O is the origin $(0, 0)$.

4. (a) Find the following indefinite integrals.

   i. $\displaystyle\int 4e^{7x} - 2\sin(3x) + \frac{1}{3x+2}\ dx$

   ii. $\displaystyle\int (x + \frac{1}{x})(x - \frac{1}{x})\ dx$

   (b) The diagram below shows parts of the graphs of the sine and cosine functions. Points C and D are chosen to lie on these graphs between their points of intersection A and B in the region shown, in such a way that the line segment CD is vertical. Show that the length of the longest such line segment CD is numerically equal to half the area enclosed between the sine and cosine curves between A and B.



5. (a) Prove that $\cos^4 x - \sin^4 x = \cos 2x$.

   (b) Water is dripping from a hole in the base of a cylinder of radius r cm, where the water height is h cm, at a rate of 0.3 $\text{cm}^3\ \text{s}^{-1}$.

      i. Fnd an expression for $\dfrac{dh}{dt}$, the rate at which the water level falls in the cylinder.

      ii. Hence find the rate of change in the water level, per minute, in a cylinder of radius 6 cm when the height of the water is 4 cm.

6. (a) Let $u = -2i$ and $v = 1 + i$, where $a, b \in \mathbb{R}$ and $i^2 = -1$, and let $z = u/v$.

      i. Write z in the form $r(\cos\theta + i\sin\theta)$, where $r, \theta \in \mathbb{R}$.

      ii. What is the angle between z and v?

      iii. Using de Moivre's theorem, or otherwise, find z^5.

   (b) Let $a, b, c, d \in \mathbb{R}$ and $z \in \mathbb{C}$. The sum of the roots of the general cubic equation $az^3 + bz^2 + cz + d = 0$ is $-b/a$, while the product of the roots is $-d/a$.

      i. Show that $z_1 = -1$ is a root of $z^3 + z^2 + z + 1 = 0$.

      ii. For the remaining roots z_2 and z_3, show that $z_2 = -z_3$.

      iii. Hence or otherwise show that $z_1^2 + z_2^2 + z_3^2 = -1$.

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Second Paper

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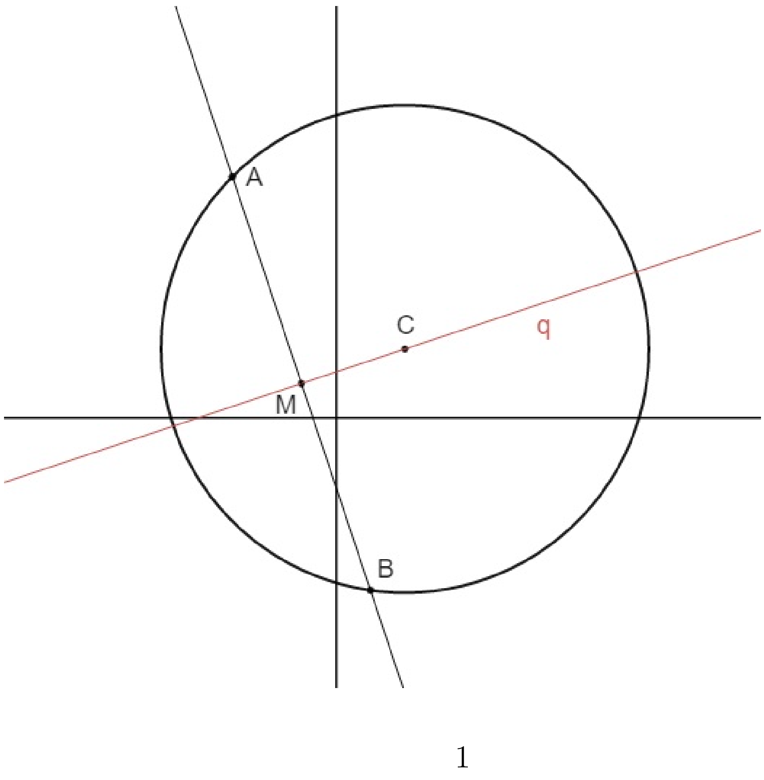
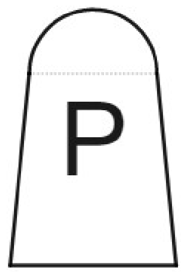
1. (a) Consider the circles $x^2 + y^2 - 2x - 2y - 7 = 0$ and $x^2 + y^2 - 8x - 10y + 37 = 0$.
   i. Show that these circles touch;
   ii. Derive the equation of their common tangent through their point of intersection;
   iii. The circles have two other common tangents; find the equation of either one of them.

   (b) The circle shown is centred at C; points A and B lie on the circle, and point M is the midpoint of the line segment AB. A has coordinates $(-3, 7)$ and M has coordinates $(-1, 1)$. Line q passes through both M and C.
   i. Determine the equation of the line q;
   ii. The coordinates of C are $(2, 2)$; find the equation of the circle.



2. (a) i. Find the two values of x satisfying $e^{2x} - 5e^x + 6 = 0$.
   ii. Find the set of values of x for which $|2x - 1| > x$.

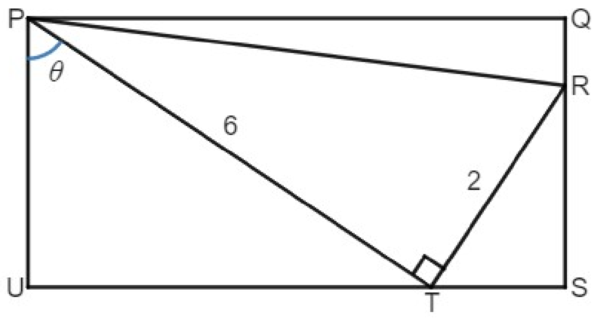
(b) A metal pepper pot is modelled as a frustum of a cone with a hemisphere on top; the radius of the hemisphere is equal to the smaller radius of the frustum.



   i. If the pepperpot has smaller and larger radii 2cm and 1cm respectively, and its total height is 4cm, what is its volume (assume it's made of negligibly thin metal).
   ii. If the pot is made of a metal that weighs 1g per square centimetre, what is its total mass (ignore any holes in the pot).

3. (a) If $\cos B = \dfrac{4t}{t^2+4}$, show that $\sin B = \dfrac{t^2-4}{t^2+4}$.

(b) The figure below shows a rectangle $PQSU$ containing a triangle PTR, right-angled at T. Given that PT has length 6, TR is of length 2 and angle UPT is denoted by θ,
   i. show that the length of the side US can be written $|US| = 6\sin\theta + 2\cos\theta$;
   ii. find a similar expression for $|RQ|$;
   iii. show that the area A of triangle PQR can be written $A = 8\sin 2\theta + 6\cos 2\theta$.



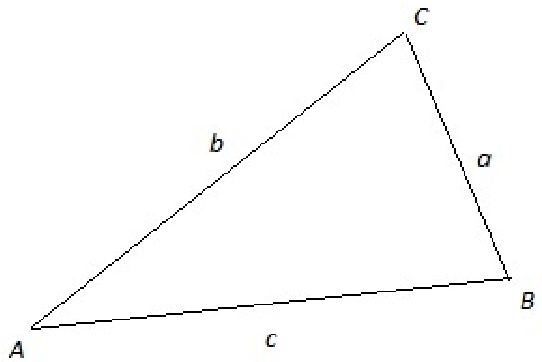
4. (a) How many arrangements are there of the letters FEDORA:
   i. if there are no restrictions,
   ii. if OR are side by side, in that order,
   iii. if O and R are separated,

iv. if vowels are grouped together?

If four letters are chosen at random from the word FEDORA without replacement, what is the probability that they will spell out the word ROAD?

(b) There are 10 points in a plane out of which 4 are collinear. Calculate the number of quadrilaterals formed by the points as vertices (show your working).

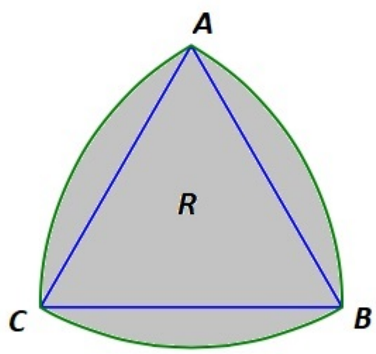
5. (a) Consider the triangle in the diagram below; find the missing sides and angles for the two possible cases satisfying $a = 3$, $b = 5$ and $\angle A = 35°$.



(b) The triangle ABC is equilateral with each side of length 6 cm. With centre A and radius 6 cm, a circular arc is drawn joining B to C. Similar arcs are drawn with centres B and C and with radii 6 cm, joining C to A and A to B respectively, as shown in the diagram. The three arcs thus drawn enclose the shaded region R.

i. Show that the area of triangle ABC is $9\sqrt{3}$ cm^2;

ii. Hence show that the area of the region R is $18(\pi - \sqrt{3})$ cm^2.



6. Each night for a week, Bobby and Boris play a game of chess.

   (a) Suppose none of the games result in a draw, and the probability on any given night that Bobby will win the game is 0.7. What is the probability that over the course of the seven days

      i. Boris wins the first game and Bobby wins the rest?
      ii. Bobby will win the the first, third, fifth and seventh games and Boris wins the rest?
      iii. Boris wins exactly five games?
      iv. Bobby's first win will happen on the fifth day?

   (b) Suppose instead that each game results in either a win or a draw, and the probability of a draw is 0.1, while the probability that Bobby will win is now 0.6.

      i. What is the probability that Boris will win?
      ii. What is the probability of three draws?
      iii. What is the probability the series is tied with three wins for each player and one draw?